# Linear Programming

For; BBA/MBA

## Linear Programming:

### 1. Introduction

Linear Programming is one of the most frequently used operations Research Technique(s). It is used to allocate scarce resources in an optimal way so that the allocator can optimize the results in terms of maximization of profits or; Minimization of Costs. Although initially this technique was applied in busines to solve Problems relating to assignment and transportation.

The Nobel laureate, George B. Dantzig innovated the algebraic technique of Solving linear programming Problems known as "Simplex Technique" in 1947.

## 2. Meaning of linear Programming

Linear Programming is a technique which is used to find the optimum (Max$^m$ or; Min$^m$) solution of a linear function of two or; more variables subject to a set of linear Constraints or; restrictions as well as a non-negativity restriction.

The Problem of optimizing (i.e Maximizing or Minimizing) a linear function, say,

$Z = Px + qy$ (P and q are Constants) Subject to a set of linear restrictions (restraints or; Constraints) as well as a non-negativity restriction is called a linear Programming Problem abbreviated as LPP.

The function $Z = Px + qy$ (P & q are Constants) to be optimized in a linear Programming Problem is called the objective function.

The word 'linear' means that the relationships involved among two or; more variables in a particular problem linear implying that the variables change proportionally.

The word 'Programming' means Planning and it refers to the Procen of determining a particular Plan of action from amongst Several alternatives.

The Variables involved in a linear Programming Problems are called "decision Variables" and the inequalities in a linear Programming Problem are Called Constraints.

## Basic Assumption(s)

Conditions of Certainity exist, that is, the Coefficients in the objective fenction and in the Constraints

(linear equations) are known with certainty and do not change during the period being studied.

Assume that;
The proportionality exists in the objective function and in the Constraints.

Additivity of all activities, which means that the total activities equal the sum of each individual activity.

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# Linear function(s) & linear Equations

Linear function in one variable is of form $ax + b \ (a \neq 0)$;

linear function in two variables is of the form;

$$ax + by + c$$

$$(a \neq 0, b \neq 0).$$

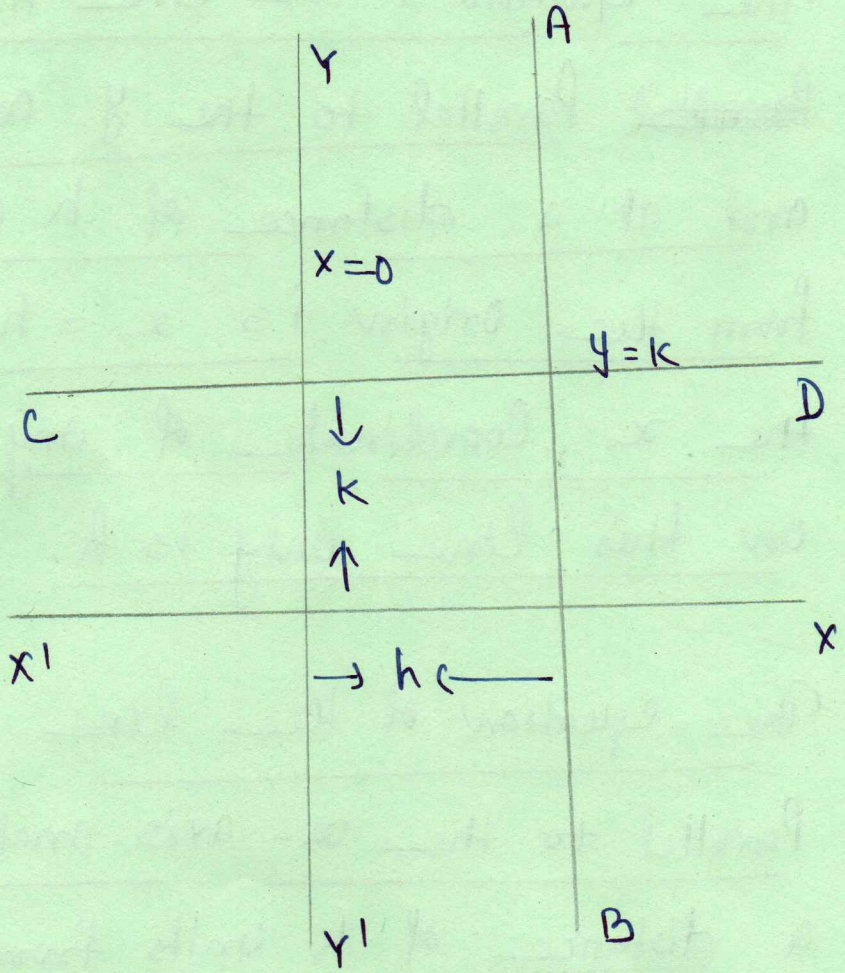
A linear function in three variables is of the form;

$$ax + by + cz + d$$

$$(a \neq 0, b \neq 0, c \neq 0)$$

And so on.

If the constant term is zero then the linear function is called a homogenous linear function. When a linear function is equated to a constant then it is called a linear Equation. A linear Equation of the form $ax + by + c = 0$,




(i) The equation of the x-axis is $y=0$, Since the y Coordinate of any point on the x-axis only is zero.

(ii) The equation of the y-axis is $x=0$, Since the x Coordinate of any point on the y-axis only is Zero.

III) The equation of the line AB ~~Parallel~~ Parallel to the y axis and at a distance of h units from the origin is $x = h$, since the x Coordinate of any point on this line only is h.

IV) The equation of the line CD Parallel to the x-axis and at a distance of k units from the origin is $y = k$, since the y Coordinate of any point on this line only is k.

## Graphical Method

Linear Programming Problems involving only two variables can be effectively solved by a graphical technique which provides a pictorial representation of the solution and one we gets insight into the basic Concepts used in~~to~~ the solving large L.P.P.

## Working Procedure:

To solve a linear Programming Problem graphically;

# First we formulate the given Problem

as a linear Programming Problem.

# Plot the given Constraints as equalities on $x_1 - x_2$ Coordinate Plane and determine the region. (convex).

# Draw the dotted line through the origin representing the objective function with $Z = 0$.

As Z is increased from Zero this line moves to the right remaining Parallel to itself.

Suppose that a firm Produces two goods X and Y, with inputs A and B. The total quantities of inputs A and B available Per Unit of time is as;

A = 1600

B = 2000

Assume that Producing one Unit of X requires 4 Units of A and 2 Units of B, and one Unit of Y requires 2 Units of A and 5 Units of B. Profits Per Unit of X and Y are estimated at Rs. 10 and Rs 8; respectively.

The overall Production Conditions may be Summed up as;

| Inputs | Total Inputs available Per time Unit | Input Requirement Per Unit of Product | |
|---|---|---|---|
| | | X | Y |
| A | 1600 | 4 | 2 |
| B | 2000 | 2 | 5 |

## (I) Objective Function:

$$\text{Maximise } \Pi = 10x + 8y \quad -(1)$$

Where x and y denote quantities of goods x and y. Their quantities multiplied by their corresponding constant (Per Unit Profit) give the total profit.

## (II) Constraint Equations:

$$4x + 2y \leq 1600 \quad \text{---} \quad (2)$$

$$2x + 5y \leq 2000 \quad \text{---} \quad (3)$$

Where $x, y, \geq 0$

According to graphical method the first step in solving the problem is to Convert the Constraint Inequalities in Equalities. So Equation 2 now

$$4x + 2y \leq 1600 \quad \text{to;}$$

$$4x + 2y = 1600$$

The terminal points can be located by assuming alternatively x and y to be zero.

If we assume

$$x = 0$$

Then $y = 800$

And If $y = 0$

Then $x = 400$

(1)

$4x + 2y = 1600$

$4.0 + 2y = 1600$

$2y = 1600$

$y = \frac{1600}{2}$

$= 800$

(2)

$4x + 2y = 1600$

$4x + 2.0 = 1600$

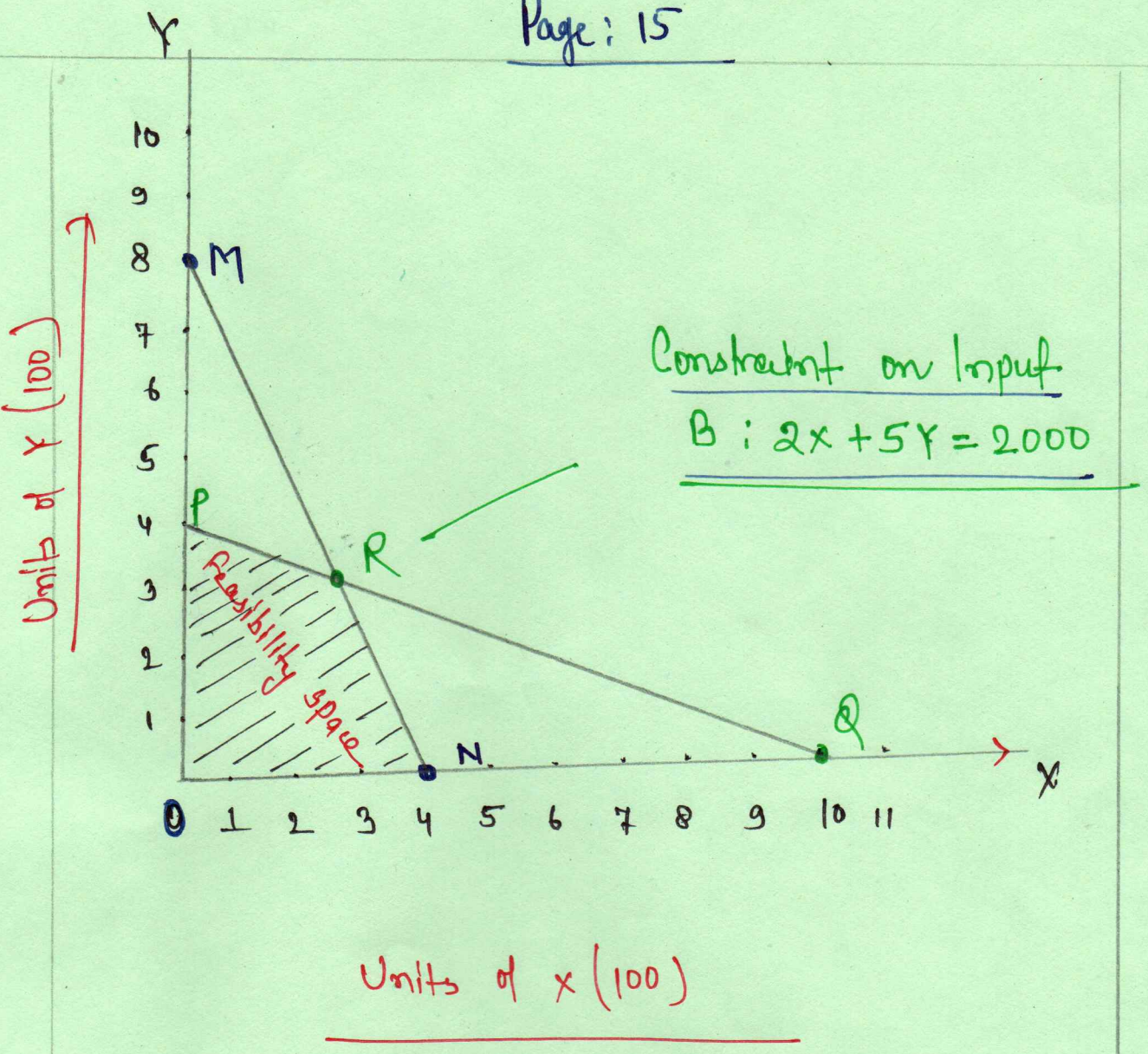
$4x = 1600$

$x = \frac{1600}{4}$

$= 400$

These two points are marked by points M and N, Respectively in equation (1). By joining these points by a Straight line, we get the Constraint line

$$4x + 2y = 1600$$




All the Points on the line MN or; to the left of it, Satisfy the Constraint

$$4x + 2y \leq 1600$$

The area OMN is the feasibility area or; feasibility space for

Single input A. It implies that any point within the feasibility space and on the border line is a feasibility point.

Note ##

The same Procedure is adopted to graph the constraint

$2x + 5y \leq 2000$ for input B, and a Constraint line PQ is obtained.

$2x + 5y = 2000$

If $x = 0$; Then $2 \times 0 + 5y = 2000$

$5y = 2000$

$y = 400$

$2x + 5y = 2000$

If $y = 0$; Then $2x + 5 \cdot 0 = 2000$

$2x = 2000$

$x = 1000$

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